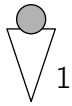
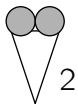
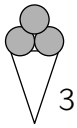
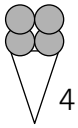
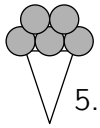
Year 9 and 10 (ENGLISH VERSION)

Thursday, 17th March 2022 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

A1 At the ice cream stand in the park, the prices for one up to five scoops of ice cream are displayed on a board. For which choice is the prize per scoop the lowest?

(**A**)  1.40 € (**B**)  2.60 € (**C**)  3.70 € (**D**)  4.60 € (**E**)  5.50 €

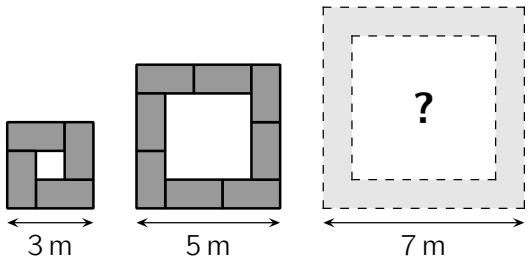
A2 An equilateral triangle and a square have the same perimeter. The equilateral triangle has side-length 12 cm. What is the side-length of the square?

(**A**) 7 cm (**B**) 8 cm (**C**) 9 cm (**D**) 10 cm (**E**) 11 cm

A3 $\dfrac{20\times 22}{(2+0)\times(2+2)} =$

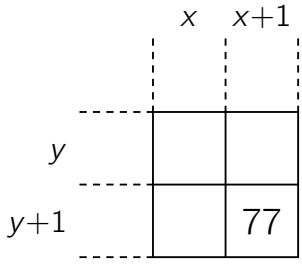
(**A**) 2 (**B**) 10 (**C**) 24 (**D**) 40 (**E**) 55

A4 In a conference room, all tables  are 2 m long and 1 m wide. They can be arranged into squares like the ones in the diagram and now need to be arranged into a 7 m × 7 m square for a meeting. How many tables are needed for this?



(**A**) 10 (**B**) 11 (**C**) 12 (**D**) 14 (**E**) 16

A5 From a multiplication table only one number can be seen, but the value of x can still be determined, since it is known that x and y are natural numbers and that $x > y > 0$. What is the value of x ?

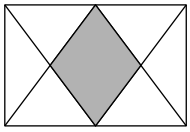


(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

A6 Which number has the property that it is less than its half, greater than its double, and that the sum of this number and its square is zero?

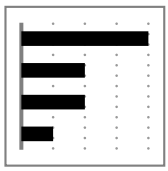
(**A**) -2 (**B**) -1 (**C**) 0 (**D**) 1 (**E**) 2

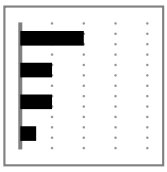
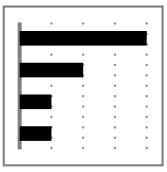
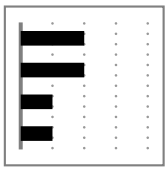
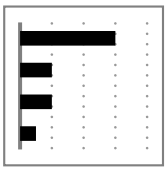
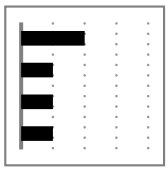
A7 In the rectangle shown, the midpoins of the two longer sides are connected to all four vertices. What fraction of the area of the rectangle is shaded?



(**A**) $\frac{1}{4}$ (**B**) $\frac{2}{7}$ (**C**) $\frac{1}{5}$ (**D**) $\frac{1}{3}$ (**E**) $\frac{2}{5}$

A8 On Nadja's smartphone, a diagram shows how much time she spent on four of her apps last week. This week, she halved the amount of time spent on two of these apps, and she spent the same amount of time on the other two of these apps. Which of the following could be the diagram for this week?



(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

A9 Once I met the headmaster's 6 daughters whose ages are 6 consecutive natural numbers. I asked each of the 6 sisters: "How old is your oldest sister?" Then, I added the 6 numbers from their answers. Which of the following numbers is definitely not the sum that I got?

(**A**) 95 (**B**) 125 (**C**) 167 (**D**) 205 (**E**) 233

A10 Five students applied for the position of student representative. Whoever receives the largest number of votes wins the election. After 90 % of the votes cast have been counted, the distribution of votes is as follows:

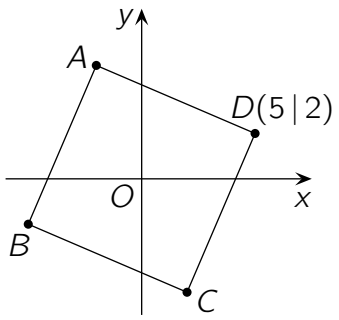
Sarah	Marco	Olina	Knut	Elias	
56	44	40	32	8	(abstentions: 0)

How many of these five students could get the largest number of votes in this election?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

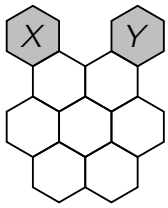
4 point problems

B1 The intersection of the diagonals of the square shown on the right is at point $O(0\,|\,0)$. The coordinates of the vertex D are $(5\,|\,2)$. What is the sum of the four coordinates of the points A and B?



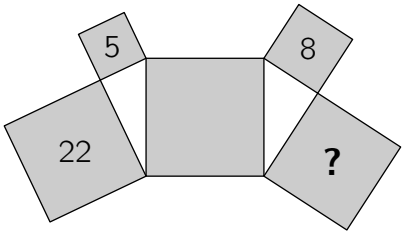
(**A**) -7 (**B**) -4 (**C**) 0 (**D**) 3 (**E**) 7

B2 In the honeycomb shown on the right, the queen bee lays an egg in the cell X. Now, she keeps crawling from one cell to a neighbouring one and lays an egg there until she reaches Y. She must lay exactly one egg in each cell. How many different paths from X to Y are possible?



(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

B3 Five squares and two right-angled triangles are arranged as shown. The three numbers 22, 5 and 8 inside three of the squares indicate their areas in m^2. What is the area of the square with the question mark?

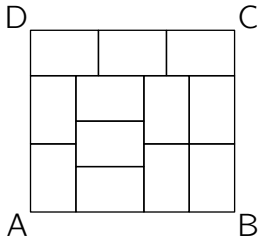


(**A**) $16\,m^2$ (**B**) $17\,m^2$ (**C**) $18\,m^2$ (**D**) $19\,m^2$ (**E**) $20\,m^2$

B4 "I dreamed that I had to mark 2022 tests," our math teacher told us. "First I marked every 6th test. When I was finished, my wife came and helped me by marking every 5th test from the remaining ones. When she was done, I had recovered and continued with marking every 4th test from the remaining ones. After that I woke up," he said. How many more tests remained to be marked?

(**A**) 337 (**B**) 674 (**C**) 1011 (**D**) 1348 (**E**) 1685

B5 The rectangle $ABCD$ in the diagram is made from 12 identical rectangles. What is the ratio of the length of the side $\overline{AD}$ to the length of the side $\overline{DC}$?

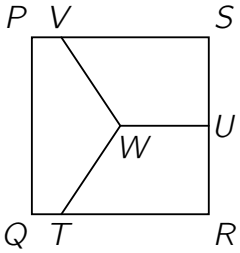


(**A**) 8 : 9 (**B**) 5 : 6 (**C**) 7 : 8 (**D**) 2 : 3 (**E**) 6 : 7

B6 The hare and the hedgehog run a race on the 440 m long outer track of a stadium. The speed of the hare is 10 m/s, and the speed of the hedgehog is 1 m/s. They start running from the starting line at the same time but in opposite directions. When they meet again, the hedgehog turns around and runs after the hare. How much later than the hare is the hedgehog back at the starting line?

(**A**) 32 s (**B**) 33 s (**C**) 35 s (**D**) 36 s (**E**) 39 s

B7 The square $PQRS$ has side-length 1 cm. U is the midpoint of the side $\overline{RS}$, and W is the centre of $PQRS$. The points V and T lie on the sides $\overline{PS}$ and $\overline{QR}$, respectively, in such a way that the areas of the three resulting regions are equal. What is the length of the line segment $\overline{SV}$?

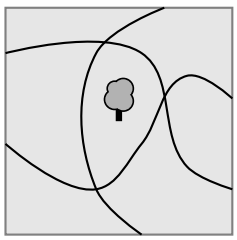


(**A**) $\frac{1}{2}$ cm (**B**) $\frac{2}{3}$ cm (**C**) $\frac{3}{4}$ cm (**D**) $\frac{4}{5}$ cm (**E**) $\frac{5}{6}$ cm

B8 When grandmother had a visit from her three grandchildren, they wanted to know how old she was. "What do you guess", grandmother asked. Each grandchild guessed a different age: 75, 78 and 81. No one was right. One had misjudged by 1 year, one by 2 years and one by 4 years. Then, the grandmother's age is

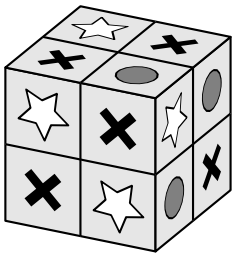
(**A**) definitely 76. (**B**) definitely 77. (**C**) definitely 79. (**D**) definitely 80.
(**E**) not uniquely determined with this information.

B9 There are three paths through our city park and there is one tree in the middle. What is the smallest number of trees that would have to be planted so that there are the same number of trees on both sides of each path?



(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

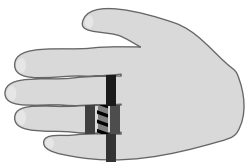
B10 Each side of the cube shown is divided into four small squares. Each small square has a sticker on it, either a cross, a star or a circle. There are always different stickers on every two small squares that have a common edge. What is the total number of circles on this cube?



(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 11

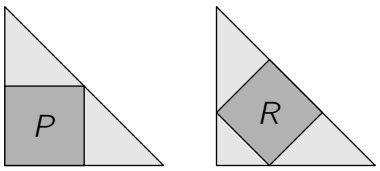
5 point problems

C1 Lilou wears 5 rings on one hand, as shown. Every now and then she takes all the rings off one by one. In how many different ways can she do this?



(**A**) 16 (**B**) 20 (**C**) 24 (**D**) 30 (**E**) 45

C2 The squares P and R are inscribed into two congruent isosceles right-angled triangles (see figure). The area of P is 45 cm^2. What is the area of R?



(**A**) 40 cm^2 (**B**) 42 cm^2 (**C**) 44 cm^2 (**D**) 45 cm^2 (**E**) 48 cm^2

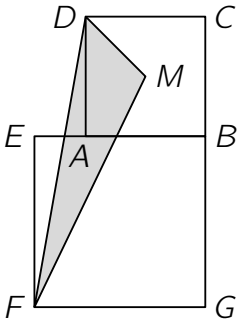
C3 Firas split the numbers from 1 to 12 into three groups of four numbers each. The sum of the numbers in the first group is 41, and the sum of the numbers in the second group is 26. Which of the following numbers is in the same group as the number 9?

(**A**) 3 (**B**) 5 (**C**) 7 (**D**) 8 (**E**) 10

C4 This year, the best football teams from eight schools will contend for the "Mayor's Cup". Each team plays against each other team exactly once. There are 3 points for the winner, 0 points for the loser and in case of a draw 1 point for each of the two teams. A total of 61 points has been awarded in the tournament. What is the largest number of points that the winning team could have?

(**A**) 16 (**B**) 17 (**C**) 18 (**D**) 19 (**E**) 21

C5 The lengths of the diagonals of the squares $ABCD$ and $EFGB$ are 7 cm and 10 cm, respectively. The point M is the intersection of the diagonals of the square $ABCD$. What is the area of the triangle FMD?



(**A**) 14.5 cm^2 (**B**) 15 cm^2 (**C**) 15.75 cm^2 (**D**) 16.5 cm^2 (**E**) 17.5 cm^2

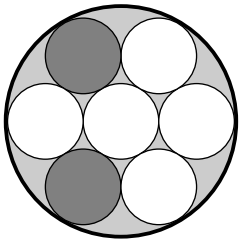
C6 The product of the digits of the number N is 20.
Which of the following numbers cannot be the product of the digits of $N + 1$?

(**A**) 40 (**B**) 35 (**C**) 30 (**D**) 25 (**E**) 24

C7 The Nature Club in our community had organised a reforestation project, 600 birch trees and 200 copper beeches were to be planted. Many volunteers took part. Each adult club member planted 10 birch trees and 5 copper beeches, every other adult planted 8 birch trees and 3 copper beeches, and each child planted 6 birch trees and one copper beech. How many people took part in the project?

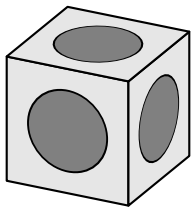
(**A**) 50 (**B**) 60 (**C**) 72 (**D**) 80 (**E**) 90

C8 Denise has a flat round tin in which she keeps her table tennis balls. Seven balls fit exactly into the tin. Two of her balls are orange, the others are white. What is the probability that the orange balls will touch each other after Denise puts the balls into the tin by chance?



(**A**) $\frac{1}{2}$ (**B**) $\frac{2}{7}$ (**C**) $\frac{5}{14}$ (**D**) $\frac{2}{3}$ (**E**) $\frac{4}{7}$

C9 In a cube with side-length 2 cm, hemispherical holes of equal size have been milled in the six faces. Each hole has exactly one point of contact with each of the four adjacent holes. The centres of the circles formed by these hemispherical holes in the faces are the centres of the corresponding faces of the cube. What is the diameter of these circles?



(**A**) $\sqrt{2}$ cm (**B**) $\frac{5}{4}$ cm (**C**) $\sqrt{\frac{3}{2}}$ cm (**D**) 1 cm (**E**) $\frac{3}{2}$ cm

C10 A table has three columns. In each cell there is a number, all numbers are different from each other. In each row, the number in the 3rd column is the sum of the numbers from the first two columns. One of the rows is marked red, and one row is marked green. If the table is sorted by the 1st column in ascending order, the 5th row is red and the 6th row is green. If the table is sorted by the 2nd column in ascending order, the 5th row is green and the 6th row is red. If the table is sorted by the 3rd column in ascending order, the 1st row is red and the last row is green. How many rows does this table have?

(**A**) 10 (**B**) 11 (**C**) 12 (**D**) 14 (**E**) 16