

Year 11 to 13 (ENGLISH VERSION)

Thursday, 16th March 2023

Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

3 point problems

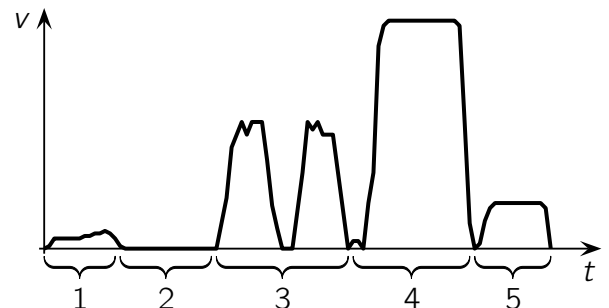
1. $\frac{77^2}{55 \times 22} =$

- (A) $\frac{49}{10}$ (B) $\frac{21}{2}$ (C) $\frac{7}{52}$ (D) $\frac{77}{16}$ (E) $\frac{14}{5}$

2. Friederike rolls five normal game dice, each with 1 to 6 pips on their sides. What is the largest number of sixes she can have rolled if there are 19 pips in total?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

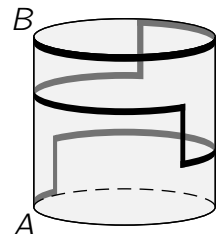
3. Neven records his way to school with his smartphone and gets the speed-time diagram shown on the right. Neven's route is divided into the sections 1 to 5. These are described in the answers. What does Neven do in section 5?



- (A) Neven rides the bus. (B) Neven runs.
 (C) Neven walks. (D) Neven waits.
 (E) Neven rides the train.

4. The cylinder shown on the right is 15 cm high and the circumference of its base is 30 cm. The path shown from A to B on the surface of the cylinder always runs vertically upwards or horizontally, i.e. parallel to the base. What is the length of this path?

- (A) 45 cm (B) 55 cm (C) 60 cm (D) 65 cm (E) 75 cm

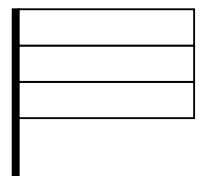


5. How many pairs (x, y) of natural numbers satisfy the equation $2x + 3y = 23$?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

6. Liane wants to colour the three stripes of the flag. She has pens in three different colours. Each stripe should be coloured with a single colour. Neighbouring stripes should have different colours. How many possibilities does Liane have for this?

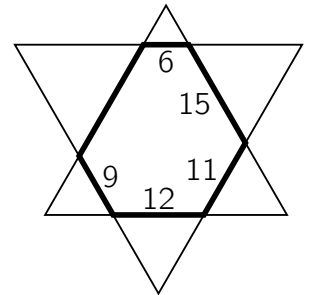
- (A) 8 (B) 10 (C) 12 (D) 16 (E) 18



7. Josephine won the jackpot of 17.1 million euros in the lottery. She did not hit the jackpot alone. The money is divided equally among all the winners. What could be Josephine's exact share?

(A) 1.1 millionen euros (B) 1.3 millionen euros (C) 1.5 millionen euros
(D) 1.7 millionen euros (E) 1.9 millionen euros

8. The two large, overlapping triangles in the picture on the right are equilateral. Opposite sides of the hexagon in the middle are parallel to each other. The lengths of five hexagon sides are given in cm. What is the length of the sixth side of the hexagon?



(A) 16 cm (B) 17 cm (C) 18 cm (D) 19 cm (E) 20 cm

9. What is the units digit of the product $(5^5 + 1) \cdot (5^6 + 1) \cdot (5^7 + 1)$?

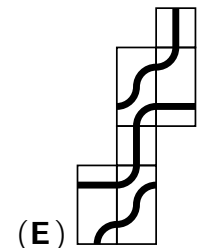
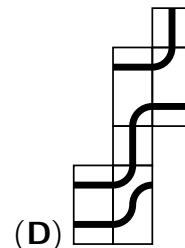
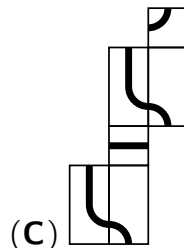
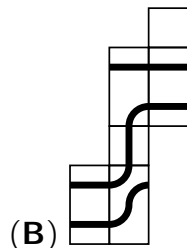
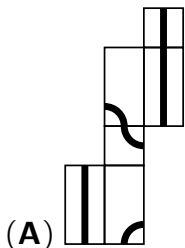
(A) 0 (B) 2 (C) 4 (D) 6 (E) 8

10. For a positive integer n , we define $n!$ (" n factorial") as the product of the numbers from 1 to n . For example, $4! = 1 \times 2 \times 3 \times 4 = 24$. For which positive integer N is $N! = 6! \times 7!$?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 14

4 point problems

11. Leonard has drawn a closed line on a cuboid. What could the net of this cuboid look like?

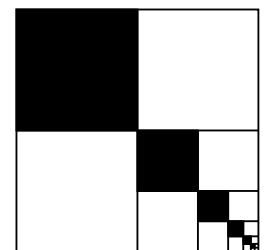


12. Medhi took lots of photos in the zoo. He printed out the 22 photos of the kangaroos and the beavers and put them in a row. Next to each photo is at least one photo of a kangaroo. What is the largest possible number of beaver photos that Medhi could have taken?

(A) 7 (B) 8 (C) 10 (D) 12 (E) 14

13. A square with area 84 cm^2 is divided into four squares and the top left square is coloured black. Then the square on the lower right is divided into four squares and of these the upper left one is coloured black. This procedure is repeated again and again. How large would the area of the black surface be if this were repeated infinitely many times?

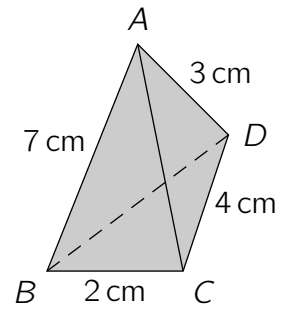
(A) 28 cm^2 (B) 31 cm^2 (C) 32 cm^2 (D) 34 cm^2 (E) 37 cm^2



14. Lucy is practising push-ups with her two sisters. Together they did 95 push-ups. Lucy's big sister did 50% more than Lucy's little sister. And Lucy did 50% more than her big sister. How many push-ups did Lucy do?

(A) 36 (B) 39 (C) 42 (D) 45 (E) 48

15. The six edges of the three-sided pyramid in the picture have integer lengths in cm. Four of the lengths are given. What is the length of the edge \overline{AC} ?



- (A) 3 cm (B) 4 cm (C) 5 cm (D) 6 cm (E) 7 cm

16. The five real numbers a_1, a_2, a_3, a_4, a_5 have sum S . We know that $a_k = k + S$ for $k = 1, 2, 3, 4, 5$. What is the value of S ?

- (A) $-\frac{15}{4}$ (B) $\frac{14}{5}$ (C) $-\frac{13}{6}$ (D) $\frac{11}{2}$ (E) $-\frac{10}{3}$

17. Before cooking, Pascal's spices are on the shelf as shown. Pascal cooks a chilli sin carne and then puts all the spices back randomly. At the end, one spice is 4 positions further to the left and another one is 3 positions further to the right than at the beginning. Also at the end, the oregano is further to the left than the ginger. Only one spice is exactly where it was at the beginning. Which one?

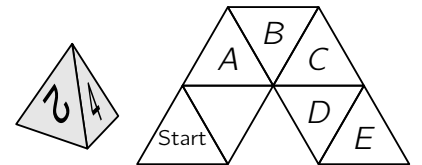


- (A) Chilli (B) Nutmeg (C) Oregano (D) Ginger (E) Pepper

18. Which of the following numbers has the same value as the number $2^{(2^{20})}$?

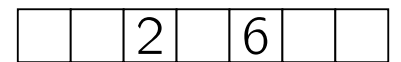
- (A) $(2^{18})^{(2^{18})}$ (B) $(2^{16})^{(2^{16})}$ (C) $(2^{12})^{(2^{12})}$ (D) $(2^{10})^{(2^{10})}$ (E) $(2^8)^{(2^8)}$

19. A tetrahedron is labelled with the numbers from 1 to 4. It is placed with the 1 on the Start field, which is shown on the right. Then it is moved from field to field by tilting it over one edge at a time. On which field does the tetrahedron lie with the 1 again for the first time?



- (A) A (B) B (C) C (D) D (E) E

20. The numbers from 1 to 7 are to be placed in the seven fields so that the sum of the numbers in three consecutive fields is always a multiple of 3. The 2 and the 6 are already placed. How many different ways are there to place the remaining 5 numbers?



- (A) 4 (B) 6 (C) 8 (D) 9 (E) 12

5 point problems

21. The numbers from 1 to 999 are arranged in ascending order in form of a spiral (see diagram). What is the arrangement of the numbers 225, 226 and 227?

:	10	11	12	13
24	9	2	3	14
23	8	1	4	15
22	7	6	5	16
21	20	19	18	17

- (A)

226	227
225	

 (B)

227
226
225

 (C)

227	
226	225

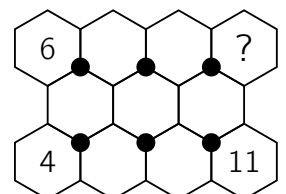
 (D)

225
226
227

 (E)

225	226	227
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22. The numbers from 1 to 11 are to be written in the hexagons. The sum of the three numbers in the hexagons around a black dot should be the same for all six dots. Three numbers have already been written in. Which number must be written in the hexagon with the question mark?



- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

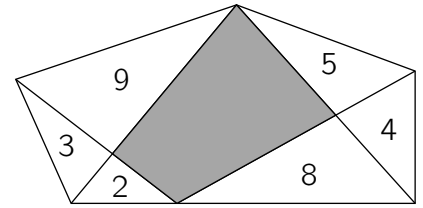
23. The point $S(x_0 | y_0)$ lies on the graphs of all functions f_a with $f_a(x) = x^3 + 4x^2 + ax + 3a + 4$, no matter which value is used for the parameter a . What is the value of y_0 ?

(A) 4 (B) 7 (C) 9 (D) 11 (E) 13

24. The large pentagon in the picture has been divided into seven parts. The numbers in the triangles indicate their area in cm^2 .

What is the area of the grey quadrilateral?

(A) 15 cm^2 (B) 16 cm^2 (C) 17 cm^2 (D) 18 cm^2 (E) 19 cm^2



25. Larissa has written a small programme. You can write a number in a field and then click a button as often as you like. Each time you click, the number in the field has its decimal part added to it. For example, 1.4365 becomes $1.4365 + 0.4365 = 1.873$ with one click and $1.873 + 0.873 = 2.746$ with another click. Oliver tries it out. He writes a number in the field and clicks the button 3 times. In the field there is now the number 10. How many different possibilities are there for the number that Oliver could have written in the field?

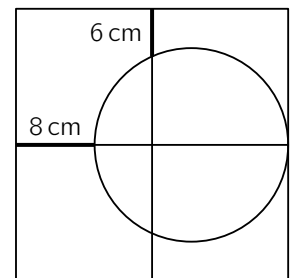
(A) 3 (B) 6 (C) 8 (D) 12 (E) 16

26. In a bouldering competition, 13 climbers compete in three categories. The overall score of each competitor is the product of her rankings in the three categories. For example, if a climber ranks 4th, 3rd and 6th, her overall score is $4 \times 3 \times 6 = 72$. The smaller the overall score, the better the overall ranking. Hannah ranks 1st in two of the categories. What is Hannah's worst possible overall ranking if no two climbers are ranked equally in any of the three categories?

(A) 2nd (B) 3rd (C) 4th (D) 5th (E) 6th

27. A large square is divided into four smaller squares. A circle inside the square touches the centre of the right side of the large square. What is the side length of the large square? (*diagram not to scale*)

(A) 18 cm (B) 20 cm (C) 24 cm (D) 28 cm (E) 30 cm



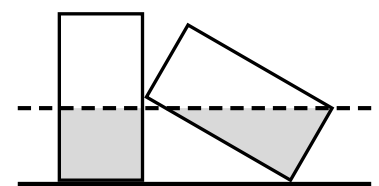
28. The two functions f and g satisfy for all real numbers x the equations $f(x) + 2g(1 - x) = x^2$ and $f(1 - x) - g(x) = x^2$. Then, $f(x) =$

(A) $x^2 - \frac{4}{3}x + \frac{2}{3}$ (B) $3x^3 + \frac{3}{2}x^2 + \frac{1}{3}$ (C) $2x^2 - \frac{1}{2}x + \frac{3}{2}$ (D) $-\frac{1}{2}x^2 + x + \frac{3}{2}$ (E) $-\frac{4}{3}x - \frac{3}{2}$

29. The 12-digit number $ABBCDDCDDABB$ is the product of 6 consecutive natural numbers. The digits A , B , C and D are consecutive, but not necessarily in this order. What is the value of the digit D ?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

30. Two identical cylindrical glasses contain the same amount of water. The area of the base of each glass is $12\pi \text{ cm}^2$. The right glass is tilted in such a way that the bottom is just completely covered with water (*see figure*). The water is at the same level in both glasses. How much water is in one glass?



(A) $54\pi \text{ cm}^3$ (B) $60\pi \text{ cm}^3$ (C) $72\pi \text{ cm}^3$ (D) $81\pi \text{ cm}^3$ (E) $96\pi \text{ cm}^3$