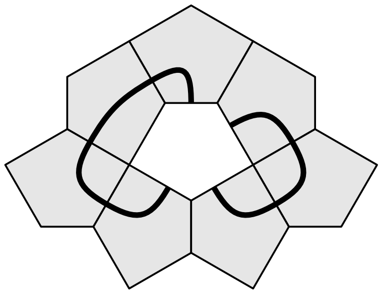
# Year 11 to 13 (English Version)

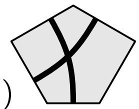
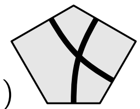
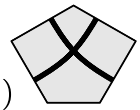
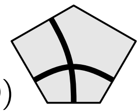
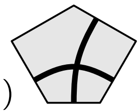
Thursday, 21st March 2024 Time allowed: 75 minutes

1. For each question exactly one of the 5 options is correct.
2. Each participant is given 30 points at the beginning. For each correct answer 3, 4 or 5 points are added. No answer means 0 points are added. If a wrong answer is given, one quarter of the points is subtracted, i. e. 0.75 points, 1 point or 1.25 points, respectively. At the end, the maximum number of points is 150, the minimum is 0.
3. Calculators and other electronic devices are not allowed.

## 3 point problems



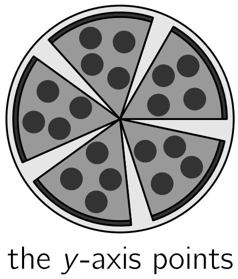
**1.** A tiling is made of equal pentagons. Which of the tiles below, when placed in the central hole, will form a self-intersecting loop?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

**2.** Which of the following numbers is 2 less than a multiple of 10, 2 greater than a square, and 2 times as large as a prime number?

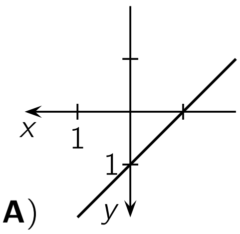
(**A**) 52 (**B**) 78 (**C**) 6 (**D**) 38 (**E**) 18

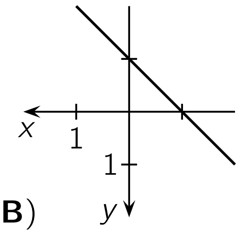
**3.** Mattis has cut a pizza into six equally sized slices. After eating one slice, he arranges the remaining slices so that the gaps between neighbouring slices are all the same size. What size is the angle between two neighbouring slices?

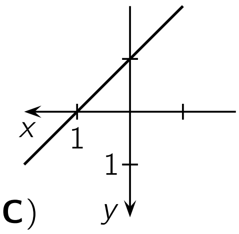


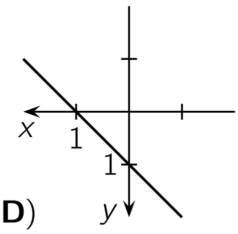
(**A**) 5° (**B**) 8° (**C**) 9° (**D**) 10° (**E**) 12°

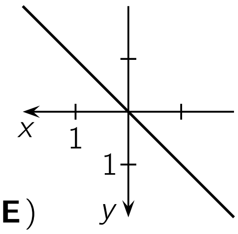
**4.** Patricia has drawn an unusual coordinate system. The $x$-axis points to the left and the $y$-axis points downwards. What does the graph of the function $f$ with $y = f(x) = x + 1$ look like in this coordinate system?

(**A**) 

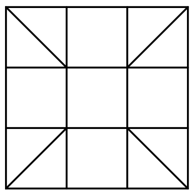
(**B**) 

(**C**) 

(**D**) 

(**E**) 


**5.** Annika wishes to colour the squares and the triangles in the diagram on the right so that no two neighbouring figures, even those sharing a single vertex, are the same colour. What is the smallest number of colours Annika needs?

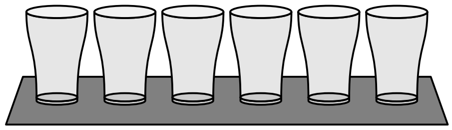


(**A**) 3 (**B**) 4 (**C**) 5 (**D**) 6 (**E**) 7

**6.** Kaito has manipulated a die. The probabilities of rolling a 2, 3, 4 or 5 are still $\frac{1}{6}$ each, but the probability of rolling a 6 is twice the probability of rolling a 1. What is the probability of rolling a 6?

(**A**) $\frac{1}{4}$ (**B**) $\frac{7}{36}$ (**C**) $\frac{1}{5}$ (**D**) $\frac{5}{18}$ (**E**) $\frac{2}{9}$

**7.** Eren puts 6 glasses on the table facing upwards, as shown. In any one move, he chooses exactly 4 glasses and turns them upside down. What is the smallest number of moves Eren needs, to have all glasses on the table facing downwards?



(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

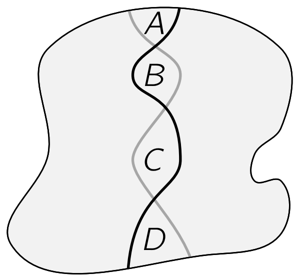
**8.** $16^{15} + 16^{15} + 16^{15} + 16^{15} =$

(**A**) $4^{19}$ (**B**) $4^{23}$ (**C**) $4^{31}$ (**D**) $4^{46}$ (**E**) $4^{60}$

**9.** Nora, Michelle and Pauline are triplets. Their teacher wants to know: "Which one of you is the oldest?" Nora answers: "I'm not the oldest." Michelle replies: "I am the oldest." Pauline answers: "I'm not the youngest." The three of them were joking, only one of them was telling the truth. In which order were the three girls born?

(**A**) Nora, Michelle, Pauline (**B**) Michelle, Nora, Pauline (**C**) Pauline, Nora, Michelle
(**D**) Pauline, Michelle, Nora (**E**) Nora, Pauline, Michelle

**10.** Both the black and the grey line divides the region shown into two equal parts. $A$, $B$, $C$ and $D$ are the areas of the regions enclosed by the lines as shown. Which statement is definitely correct?



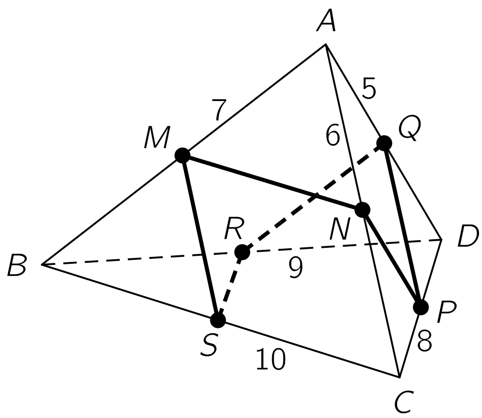

(**A**) $A + D = B + C$ (**B**) $A = D$ (**C**) $C = A + B + D$
(**D**) $A + C = B + D$ (**E**) $A + B = C + D$

**4 point problems**

**11.** Peter has many black and many white unit cubes. He wants to build a $3 \times 3 \times 3$ cube using 27 of them. Exactly half the surface of this cube should be black. What is the smallest number of black cubes Peter needs for this?

(**A**) 9 (**B**) 10 (**C**) 11 (**D**) 12 (**E**) 13

**12.** The pyramid $ABCD$ has the edges of length $|AD| = 5\,\text{cm}$, $|AC| = 6\,\text{cm}$, $|AB| = 7\,\text{cm}$, $|CD| = 8\,\text{cm}$, $|BD| = 9\,\text{cm}$, $|BC| = 10\,\text{cm}$. The points $M$, $N$, $P$, $Q$, $R$ and $S$ are the midpoints of the edges. How long is the closed path $MNPQRSM$?




(**A**) 19 cm (**B**) 20 cm (**C**) 21 cm (**D**) 22 cm (**E**) 23 cm

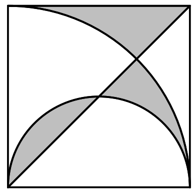
**13.** Given is a natural number $n$. Exactly one of the following statements about $n$ is true, the other four are false. Which statement is true?

(**A**) $n$ is divisible by 3 (**B**) $n$ is divisible by 6
(**C**) $n$ is a prime number (**D**) $n = 2$ (**E**) $n$ is odd

**14.** Seyma has written down the numbers 6 and 15 several times and then calculated their product. One of the following multiplications also gives the result that Seyma obtained. Which one?

(**A**) $2^8 \times 3^8 \times 5^8$ (**B**) $2^4 \times 3^6 \times 5^{10}$ (**C**) $2^6 \times 3^{10} \times 5^8$ (**D**) $2^7 \times 3^{12} \times 5^5$ (**E**) $2^5 \times 3^{15} \times 5^3$

**15.** A diagonal, a semicircle and a quarter circle have been drawn in a square with side length 6 cm as shown. What is the total area of the grey part?



(**A**) $12\,\text{cm}^2$ (**B**) $\frac{10}{3}\pi\,\text{cm}^2$ (**C**) $3\pi\,\text{cm}^2$ (**D**) $9\,\text{cm}^2$ (**E**) $(6\pi-9)\,\text{cm}^2$

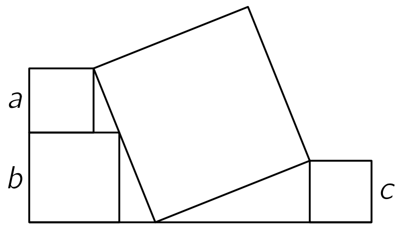
**16.** We have two positive numbers $p$ and $q$, with $p < q$. Which of the following fractions has the largest value?

(**A**) $\frac{p+3q}{4}$ (**B**) $\frac{p+2q}{3}$ (**C**) $\frac{p+q}{2}$ (**D**) $\frac{2p+q}{3}$ (**E**) $\frac{3p+q}{4}$

**17.** Zuzanna has collected porcini mushrooms and wants to dry them in the oven at a low temperature. Fresh porcini mushrooms consist of 90 % water. After some time in the oven, the water only makes up 20 % of the mass. By what percentage has the mass of the mushrooms decreased?

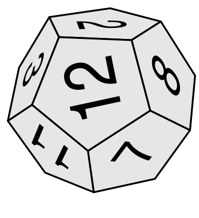
(**A**) by 72.5 % (**B**) by 75 % (**C**) by 85 % (**D**) by 87.5 % (**E**) by 90 %

**18.** Four squares are shown on the right. The side lengths of the three small squares are $a$, $b$ and $c$ respectively. What is the side length of the large square?




(**A**) $\sqrt{(a+b)^2+c^2}$ (**B**) $\frac{1}{2}(a+b+c)$ (**C**) $\sqrt{a^2+b^2+c^2}$
(**D**) $\sqrt{ab+bc+ac}$ (**E**) $\sqrt{(b-a)^2+c^2}$

**19.** Henriette has several 12-sided playing dice. The sides are labelled with the numbers from 1 to 12. When rolling all the dice at once, the probability of rolling a 12 exactly once is equal to the probability of rolling no 12 at all. How many 12-sided dice does Henriette have?



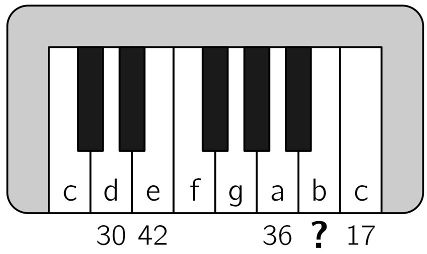
(**A**) 5 (**B**) 8 (**C**) 11 (**D**) 18 (**E**) 23

**20.** If you place a decimal point between $q$ and $r$ in the 4-digit number $N = \overline{pqrs}$, you get the average (arithmetic mean) of the two 2-digit numbers $\overline{pq}$ and $\overline{rs}$. What is the sum of the digits of $N$?

(**A**) 14 (**B**) 18 (**C**) 21 (**D**) 25 (**E**) 27

## 5 point problems

**21.** Joseph has given his niece a children's piano for her second birthday. She tries it out straight away and hits the keys again and again with her whole hand. This way she always presses 4 neighbouring white keys at once. She presses the d a total of 30 times, the e 42 times, the a 36 times and the high c 17 times. How many times did she press the b?




(**A**) 19 times (**B**) 24 times (**C**) 27 times (**D**) 32 times (**E**) 35 times

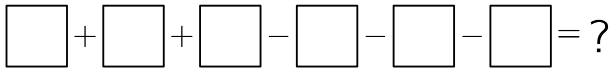
**22.** Consider three different non-zero integers $a$, $b$ and $c$. For the real number $x$ both $ax^2+bx+c=0$ and $bx^2+ax+c=0$ are true. Which of the following statements is then certainly true?

(**A**) $a+b+c=0$ (**B**) $2bc=a^2$ (**C**) $ac=b$ (**D**) $a^2-b^2=c^2$ (**E**) $ab=c$

**23.** Two candles of the same height have different thicknesses. They are lit at the same time and burn evenly. One candle burns down completely in exactly 5 hours and the other one in 4 hours. For how long must the two candles burn until one candle is 3 times as high as the other?

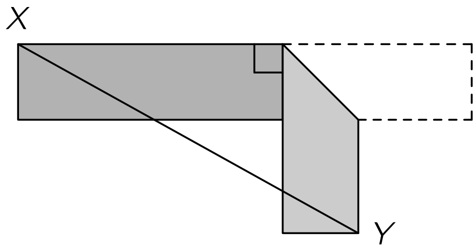
(**A**) $\frac{40}{11}$ hours (**B**) $\frac{45}{12}$ hours (**C**) $\frac{63}{20}$ hours (**D**) $\frac{54}{17}$ hours (**E**) $\frac{47}{14}$ hours

**24.** Tristan has six cards, each with a number on the front and a number on the back. The pairs of numbers on the six cards are $(5, 12)$, $(3, 11)$, $(0, 16)$, $(7, 8)$, $(4, 14)$ and $(9, 10)$. Tristan places these six cards on the empty spaces shown. What is the smallest value this calculation can yield?




(**A**) $-28$ (**B**) $-27$ (**C**) $-26$ (**D**) $-25$ (**E**) $-24$

**25.** A rectangular strip of paper is 12 cm long and 2 cm wide. The right end is to be folded at one point so that it points vertically downwards. What is the smallest possible length of the segment $\overline{XY}$?




(**A**) $6\sqrt{2}$ cm (**B**) $7\sqrt{2}$ cm (**C**) 10 cm
(**D**) 8 cm (**E**) $(6 + \sqrt{2})$ cm

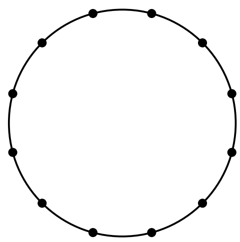
**26.** The four-digit number $\overline{abcd}$ satisfies the equation $\overline{abcd} = a^a + b^b + c^c + d^d$. What is the value of $a$?

(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

**27.** The polynomial function $p$ with $p(x) = ax^2 + bx + c$ satisfies the equation $p(x+1) = x^2 - x + 2 \times p(6)$ for all real numbers $x$. Then $a + b + c =$

(**A**) $-40$ (**B**) $-36$ (**C**) $-6$ (**D**) 12 (**E**) 40

**28.** Twelve points were marked on a circle dividing it up into 12 arcs of equal length. How many triangles are there whose vertices are 3 of the given points and which have at least one interior angle that is 45°?



(**A**) 48 (**B**) 60 (**C**) 72 (**D**) 84 (**E**) 96

**29.** The real numbers $x, y, z$ satisfy $2^x = 3$ and $2^y = 7$ and $6^z = 7$. How can $z$ be calculated using $x$ and $y$?

(**A**) $z = \frac{x}{y} + 1$ (**B**) $z = \frac{y}{x} - 1$ (**C**) $z = \frac{x}{y-1}$ (**D**) $z = y - \frac{1}{x}$ (**E**) $z = \frac{y}{1+x}$

**30.** Emily and Daniel are playing a game. They write the natural numbers from 1 to 7 on a piece of paper. Then, they take it in turns to choose a remaining number and cross it out and also all its divisors. Whoever crosses out the last number wins. Emily starts and Daniel uses every opportunity to win the game. Which number must Emily choose on her first turn so that she can win the game?

(**A**) 5 or 7 (**B**) 4 (**C**) 3 (**D**) 2 (**E**) 1